RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced GCE

Unit **4763:** Mechanics 3

Mark Scheme for January 2011

Oxford Cambridge and RSA Examinations

1(a)(i)	$[\text{ Force }] = \text{M L T}^{-2}$ $[\text{ Density }] = \text{M L}^{-3}$ $[\text{ Angular speed }] = \text{T}^{-1}$	B1 B1 B1 **3**	*Deduct one mark if given as* kg m s^{-2} *etc*
(ii)	$[\text{ Breaking stress }] = \dfrac{\text{M L T}^{-2}}{\text{L}^2}$ $= \text{M L}^{-1}\text{T}^{-2}$	M1 E1 **2**	For $[\text{ Force }] \div \text{L}^2$
(iii)	$1.2\times10^9 \times \dfrac{1}{0.454} \times 0.0254 \times 0.001^2$ $= 67.1 \text{ lb in}^{-1}\text{ ms}^{-2}$ (3 sf)	M1 M1 A1 **3**	For $\times 0.001^2$ or $\times \dfrac{1}{0.001^2}$ For $\times \dfrac{0.0254}{0.454}$ 5.6×10^{-8} implies M1M1A0 2.15×10^{16} implies M1M0A0
(iv)	$\text{T}^{-1} = (\text{M L}^{-1}\text{T}^{-2})^{\alpha}(\text{M L}^{-3})^{\beta}\text{L}^{\gamma}$ $\alpha = \frac{1}{2}$ $\beta = -\frac{1}{2}$ $0 = -\alpha - 3\beta + \gamma$ $\gamma = -1$	B1 B1 M1 A1 **4**	*All marks ft provided work is comparable* Considering powers of L
(v)	$3140 = k(1.2\times10^9)^{\frac{1}{2}}(7800)^{-\frac{1}{2}}(0.5)^{-1}$ $k = 4.00$ (3 sf) $S = \dfrac{\omega^2\rho r^2}{k^2} = \dfrac{8120^2 \times 2700 \times 0.2^2}{4^2}$ $= 4.44\times10^8$ Pa (3 sf)	M1 M1 M1 A1 cao **4**	Obtaining equation for *k* Obtaining numerical value for *k* Obtaining equation for *S*
	OR $S = 1.2\times10^9 \times \left(\dfrac{8120}{3140}\right)^2 \times \dfrac{2700}{7800} \times \left(\dfrac{0.2}{0.5}\right)^2$ M1M1M1 $= 4.44\times10^8$ A1		
(vi)	$\omega = 4(630)^{\frac{1}{2}}(70)^{-\frac{1}{2}}(15)^{-1}$ $= 0.8$	M1 A1 cao **2**	Obtaining equation for ω

2(a)(i)	$T\cos\alpha = m\dfrac{V^2}{r}$ (α is angle APC) $T\times\dfrac{8.4}{30} = 48\times\dfrac{3.5^2}{8.4}$ Tension is 250 N $T\sin\alpha + R = mg$ $250\times0.96 + R = 48\times9.8$ Normal reaction is 230.4 N	M1 A1 A1 M1 A1 **5**	Equation of motion including $\dfrac{V^2}{r}$ Or $T\cos73.7 = ...$ or $T\sin16.3 = ...$ Resolving vertically (three terms)
(ii)	$T\sin\alpha = mg$ $T\times0.96 = 48\times9.8$ $T = 490$ $T\cos\alpha = m\dfrac{V^2}{r}$ $490\times0.28 = 48\times\dfrac{V^2}{8.4}$ $V = 4.9$	M1 A1 M1 A1 **4**	Vertical equation with $R = 0$ Or $T\sin73.7 = ...$ or $T\cos16.3 = ...$ Obtaining equation for V *Allow T=490 obtained in (i) and used correctly in (ii) for full marks*
(b)(i)	$\tfrac{1}{2}m(v^2-u^2) = m\times9.8(2.5-2.5\cos\theta)$ $v^2-u^2 = 49(1-\cos\theta)$ $v^2 = u^2+49-49\cos\theta$	M1 A1 E1 **3**	Equation involving KE and PE
(ii)	$mg\cos\theta - R = m\dfrac{v^2}{r}$ $48\times9.8\left(\dfrac{u^2+49-v^2}{49}\right) - R = \dfrac{48v^2}{2.5}$ $9.6u^2+470.4-9.6v^2-R = 19.2v^2$ $R = 470.4+9.6u^2-28.8v^2$	M1 A1 M1 A1 **4**	Radial equation (three terms) Obtaining equation in *R*, *u*, *v*
(iii)	$470.4+9.6u^2-28.8\times4.15^2 = 0$ $u = 1.63$ (3 sf)	M1 A1 **2**	Substituting $R = 0$ and $v = 4.15$ *or other complete method leading to an equation for u* (ft requires $0 < u < 4.15$)

3 (i)	Tension is $180(10-7)$ $= 540$ N	M1 A1 **2**	Using $T = k \times$ extension
(ii)	$4 \times 540 = T + 200 \times 9.8$ $T = 200$ Extension is $\frac{200}{80}$ ($= 2.5$) Natural length is 5.5 m	M1 A1 M1 A1 cao **4**	Resolving vertically
(iii)	$80(2.5-x) + 200 \times 9.8 - 4 \times 180(3+x) = 200\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$ $200 - 80x + 1960 - 2160 - 720x = 200\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$ $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -4x$	B1 ft M1 A1 E1 **4**	For $180(3+x)$ or $80(2.5-x)$ Equation of motion (condone one missing force)
(iv)	Maximum acceleration is $\omega^2 A$ $= 4 \times 2.2 = 8.8 \text{ m s}^{-2}$	M1 A1 **2**	*Condone* -8.8
(v)	When $x = -1.6$, $v^2 = \omega^2(A^2 - x^2)$ $= 4(2.2^2 - 1.6^2)$ Speed is 3.02 m s^{-1} (3 sf)	M1 A1 **2**	Using $v^2 = \omega^2(A^2 - x^2)$ (or other complete method) *(Allow M1 if* $\omega^2 = 2$ *or* 16 *used but M0 if* $x = 3.8$ *is used)* *Condone* -3.02
(vi)	$x = 2.2\cos 2t$ When $t = 5$, $x = -1.846$ Period is $\frac{2\pi}{\omega} = \pi$, 5 s is $\frac{5}{\pi} \approx 1.6$ periods Distance travelled is $6 \times 2.2 + (2.2 - 1.846)$ $= 13.6$ m (3 sf)	B1 M1 M1 A1 **4**	*Condone* $x = -2.2\cos 2t$ *This B1 can be earned in (v)* Obtaining x when $t = 5$ (from $x = A\cos\omega t$ or $x = A\sin\omega t$) Correct strategy for finding distance

4 (a)	Volume is $\int \pi y^2 \mathrm{d}x = \int_k^{4k} \pi(x^2 - k^2)\,\mathrm{d}x$	M1	For $\int (x^2 - k^2)\,\mathrm{d}x$
	$= \pi\left[\, \frac{1}{3}x^3 - k^2 x \,\right]_k^{4k} \quad (= 18\pi k^3)$	A1	For $\frac{1}{3}x^3 - k^2 x$
	$\int \pi x y^2 \mathrm{d}x = \int_k^{4k} \pi(x^3 - k^2 x)\,\mathrm{d}x$	M1	For $\int xy^2 \mathrm{d}x$
	$= \pi\left[\, \frac{1}{4}x^4 - \frac{1}{2}k^2 x^2 \,\right]_k^{4k} \quad \left(= \frac{225\pi k^4}{4}\right)$	A1A1	For $\frac{1}{4}x^4$ and $-\frac{1}{2}k^2 x^2$
	$\bar{x} = \dfrac{\frac{225}{4}\pi k^4}{18\pi k^3}$	M1	*Dependent on previous M1M1*
	$= \dfrac{25k}{8} = 3.125k$	A1 **7**	
(b)(i)	Area is $\int_0^{2a} \dfrac{x^3}{a^2}\,\mathrm{d}x$	M1	For $\int \dfrac{x^3}{a^2}\,\mathrm{d}x$
	$= \left[\dfrac{x^4}{4a^2} \right]_0^{2a} \quad (= 4a^2)$	A1	For $\dfrac{x^4}{4a^2}$
	$\int xy\,\mathrm{d}x = \int_0^{2a} \dfrac{x^4}{a^2}\,\mathrm{d}x$	M1	For $\int xy\,\mathrm{d}x$
	$= \left[\dfrac{x^5}{5a^2} \right]_0^{2a} \quad \left(= \dfrac{32a^3}{5}\right)$	A1	For $\dfrac{x^5}{5a^2}$
	$\bar{x} = \dfrac{\frac{32}{5}a^3}{4a^2} = \dfrac{8a}{5} = 1.6a$	A1	
	$\int \frac{1}{2}y^2\,\mathrm{d}x = \int_0^{2a} \dfrac{x^6}{2a^4}\,\mathrm{d}x$	M1	For $\int y^2\,\mathrm{d}x$ or $\int (2a - x)y\,\mathrm{d}y$
	$= \left[\dfrac{x^7}{14a^4} \right]_0^{2a} \quad \left(= \dfrac{64a^3}{7}\right)$	A1	For $\dfrac{x^7}{14a^4}$ or $ay^2 - \frac{3}{7}a^{2/3}y^{7/3}$
	$\bar{y} = \dfrac{\frac{64}{7}a^3}{4a^2} = \dfrac{16a}{7}$	A1 **8**	
(ii)	Centre of mass is vertically below A	M1	*May be implied*
	$\tan\theta = \dfrac{2a - \bar{x}}{8a - \bar{y}} = \dfrac{\frac{2}{5}a}{\frac{40}{7}a} \quad (= 0.07)$	M1	Condone reciprocal
	Angle is $4.00°$ (3 sf)	A1 **3**	